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Knowledge and action: how should we combine their logics?

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Outline

1. Motivation: simple logics of action and knowledge needed
2. Adding higher-order observability information
3. Application: the gossip problem
4. Adding public announcements
5. Application: the muddy children puzzle
6. Application: boolean games
Logics of knowledge and action

- fruitful in CS since 30+ years
  - epistemic temporal logics
    [Halpern et col., Lomuscio, ... , ≥1990]
  - epistemic extension of the situation calculus
    [Scherl & Levesque, ... , ≥1995]
  - Dynamic Epistemic Logics DEL
    [van Benthem, Moss, Baltag, van Ditmarsch, ... , ≥2000]
- typically multi-dimensional modal logics
  - high complexity; often undecidable
- simplest combined logic of knowledge and action?
  - a typical question of philosophical logic
  - also relevant for computer science
Logics of knowledge and action

- idea [v.d.Hoek & Wooldridge, inspired from model checkers]:
  - ground action on propositional control
  - ground knowledge on propositional observability

- logics:

  \[
  \begin{align*}
  \text{ECL-P0} & = \text{“Epistemic Coalition Logic of Propositional Control with Partial Observability”} \ [vdHTW11] \\
  \text{LRC} & = \text{“Logic of Revelation and Concealment”} \ [vdHIW12]
  \end{align*}
  \]

- this talk:
  - reduce to \textit{Dynamic Logic of Propositional Assignments} DL-PA
  - overcome some limitations of the original approach
Grounding action on propositional control

agent $i$ controls propositional variable $p$ or not

- define accessibility relation for group of agents $J \subseteq \text{Agt}$:
  \[ R_J = \{(v, v') : v(p) = v'(p) \text{ if } p \in PVar \text{ not controlled by any } i \in J\} \]

- coalitional effectivity *ceteris paribus*:
  \[ vR_J v' = \text{at } v, \text{ if the other agents don't act then } J \text{ can guarantee that the next state of the world is } v' \]

- interpret operator of coalitional effectivity:
  \[ v \models \Diamond_J \varphi \text{ iff } v' \models \varphi \text{ for every } v' \text{ such that } vR_J v' \]

  \[ \implies \text{Coalition Logic of Propositional Control} \]

- approximates ATL/Pauly’s operator of coalitional effectivity:
  \[ \langle \langle \{i\} \rangle \rangle X \varphi \approx \Diamond \{i\} \Box \text{Agt}\backslash \{i\} \varphi \]
Grounding knowledge on propositional observability

agent $i$ observes whether propositional variable $p$ is true or not

- muddy children: child 1 sees whether child 2 is muddy; doesn’t see whether 1 is muddy
- define indistinguishability relation:

$$\sim_1 \quad \sim_2 \quad \sim_1$$

$$\{m_1\} \sim_2 \{m_1, m_2\}$$

$$\sim_1 \quad \sim_2 \quad \sim_1$$

$$\emptyset \sim_2 \{m_2\}$$

$$\sim_i = \{(v, v') : v(p) = v'(p) \text{ for every } p \in P\text{Var} \text{ observed by } i\}$$

$\Rightarrow$ equivalence relation on the set of all valuations

- interpret epistemic operator as usual:

$$v \models K_i \varphi \iff v' \models \varphi \text{ for every } v' \text{ such that } v \sim_i v'$$

- pushes the envelope of the ‘DEL philosophy’ of replacing accessibility relations by model updates

(while DELs still have accessibility relations for knowledge)
Propositional observability: properties

- all axiom schemas of S5 valid
- observability is common knowledge:
  \[(K_i p \lor K_i \neg p) \rightarrow K_j (K_i p \lor K_i \neg p)\]
  \[\neg(K_i p \lor K_i \neg p) \rightarrow K_j \neg(K_i p \lor K_i \neg p)\]
- distributes over disjunction:
  \[K_i (p \lor q) \leftrightarrow (K_i p \lor K_i q)\]

so:
- initial situation of the muddy children puzzle can be modelled
- ... but not the situation after the father’s announcement “one of you is muddy”!
- related:
  - logic only accounts for observation but not for communication
Embedding into DL-PA

- can be captured in
  *Dynamic Logic of Propositional Assignments* DL-PA

1. introduce new propositional variables
   \[ C_i \ p = \text{“} i \ \text{controls} \ p \text{”} \]
   \[ S_i \ p = \text{“} i \ \text{sees} \ p \text{”} \]

2. identify \( \Diamond_i \) and \( K_i \) with assignment programs:
   for \( \varphi \) boolean with \( PVar(\varphi) = \{ p_1, \ldots, p_n \} \),
   \[
   \Diamond_i \varphi \leftrightarrow \langle \left( \neg C_i \ p_1 ? \sqcup (C_i \ p_1 ?; (+p_1 \sqcup \neg p_1)) \right) ; \\
   \quad \cdots ; \\
   \quad \left( \neg C_i \ p_n ? \sqcup (C_i \ p_n ?; (+p_n \sqcup \neg p_n)) \right) \rangle \varphi
   \]
   \[
   K_i \varphi \leftrightarrow \left[ \left( S_i \ p_1 ? \sqcup (\neg S_i \ p_1 ?; (+p_1 \sqcup \neg p_1)) \right) ; \\
   \quad \cdots ; \\
   \quad \left( S_i \ p_n ? \sqcup (\neg S_i \ p_n ?; (+p_n \sqcup \neg p_n)) \right) \right] \varphi
   \]
   \( \implies \) start with innermost modal operators!

3. axiomatize exclusive and exhaustive control
   \[
   \left( \bigwedge \neg (C_i \ p \land C_j \ p) \right) \land \left( \bigvee C_i \ p \right)
   \]
DL-PA

- assignment programs built by the PDL program operators from
  
  \[ +p \quad = \quad \text{“make } p \text{ true”} \]
  
  \[ -p \quad = \quad \text{“make } p \text{ false”} \]

- generalizes QBF:
  
  \[ \forall p. \varphi \iff [+p \cup -p] \varphi \]

- compact models
  
  - valuations of classical propositional logic

- \text{PSPACE} complete (both model checking and SAT)

- uniform substitution does not preserve validity
Adding higher-order observability information
Higher-order observability

• idea: introduce higher-order visibility atoms

\[ S_i p = "i \text{ sees the value of } p" \]
\[ S_i S_j p = "i \text{ sees whether } j \text{ sees the value of } p" \]
\[ S_i S_j S_k p = "\ldots" \]

• general schema as before:

\[ K_i \varphi \leftrightarrow [\pi_{i,\text{Atm}(\varphi)}] \varphi \]

where \( \pi_{i,\text{Atm}(\varphi)} = (S_i \alpha_1 ? \square (\neg S_i \alpha_1 ?; (+\alpha_1 \sqcup -\alpha_1))) ; \ldots \)

examples:

\[ K_i p \leftrightarrow p \land S_i p \]
\[ K_i \neg p \leftrightarrow \neg p \land S_i p \]
\[ K_i K_j p \leftrightarrow K_i (p \land S_j p) \]
\[ \leftrightarrow K_i p \land K_i S_j p \]
\[ \leftrightarrow p \land S_i p \land S_j p \land S_i S_j p \]

DEL-PA0 = DEL of Propositional Assignment and Observation
Language of DEL-PAO

- visibility atoms:

\[ \alpha ::= p \mid S_i \alpha \mid JS \alpha \]

with \( p \) propositional variable and \( i \) agent

\[ p = \ldots \]
\[ S_i \alpha = \ldots \]
\[ JS \alpha = \text{"all agents jointly see whether } \alpha \text{"} \]

- formulas and programs as in PDL:

\[ \varphi ::= \alpha \mid \neg \varphi \mid \varphi \land \varphi \mid K_i \varphi \mid CK \varphi \mid [\pi] \varphi \]
\[ \pi ::= +\alpha \mid -\alpha \mid \pi; \pi \mid \pi \sqcup \pi \mid \varphi? \]

with \( i \) agent and \( \alpha \) visibility atom
DEL-PAO: valuations

- valuation $= \text{sets of visibility atoms } v$
- define indistinguishability relations:
  \[ v \sim_i v' \quad \text{iff} \quad \forall \alpha, \text{ if } S_i \alpha \in v \text{ then } v(\alpha) = v'(\alpha) \]
  \[ v \sim_{Agt} v' \quad \text{iff} \quad \forall \alpha, \text{ if } JS \alpha \in v \text{ then } v(\alpha) = v'(\alpha) \]
- problem: are reflexive, but neither transitive nor symmetric
  - $\emptyset \sim_i v$ for every $v$
  - $v \not\sim_i \emptyset$ as soon as $p \in v$ and $S_i p \in v$
DEL-PAO: valuations

- valuation = sets of visibility atoms $v$
- define indistinguishability relations:
  $v \sim_i v'$ iff $\forall \alpha, \text{ if } S_i \alpha \in v \text{ then } v(\alpha) = v'(\alpha)$
  $v \sim_{Agt} v'$ iff $\forall \alpha, \text{ if } JS \alpha \in v \text{ then } v(\alpha) = v'(\alpha)$
- problem: are reflexive, but neither transitive nor symmetric
  - $\emptyset \sim_i v$ for every $v$
  - $v \not\sim_i \emptyset$ as soon as $p \in v$ and $S_i p \in v$
- solution: valuations must be introspective
DEL-PAO: introspective valuations

Definition

\( \nu \) is introspective iff

1. \( S_i S_i \alpha \in \nu \)
2. \( JS JS \alpha \in \nu \)
3. \( JS S_i S_i \alpha \in \nu \)
4. if \( JS \alpha \in \nu \) then \( S_i \alpha \in \nu \)
5. if \( JS \alpha \in \nu \) then \( JS S_i \alpha \in \nu \)

Theorem

Introspective valuations contain all atoms of form “\( \cdots S_i S_i \cdots p \)” and “\( \cdots JS JS \cdots p \)”

Theorem

\( \sim_i \) and \( \sim_{Agt} \) are equivalence relations on introspective valuations
DEL-PAO: interpretation of formulas

• interpretation of formulas:
  \[ \nu \models \alpha \quad \text{iff} \quad \alpha \in \nu \]
  \[ \nu \models K_i \varphi \quad \text{iff} \quad \nu' \models \varphi \text{ for every } \nu \sim_i \nu' \]
  \[ \nu \models CK \varphi \quad \text{iff} \quad \nu' \models \varphi \text{ for every } \nu \sim_{Agt} \nu' \]
  \[ \nu \models [\pi] \varphi \quad \text{iff} \quad \nu' \models \varphi \text{ for every } \nu R_\pi \nu' \]

• interpretation of programs:
  \[ \nu R_+\alpha \nu' \quad \text{iff} \quad \nu' = \nu \cup \{ \alpha \text{ and its introspective consequences} \} \]
  \[ \nu R_-\alpha \nu' \quad \text{iff} \quad \alpha \text{ is not an introspectively valid atom} \]
  \[ \quad \text{and } \nu' = \nu \setminus \{ \alpha \text{ and its causes} \} \]
  \[ \nu R_{\pi_1;\pi_2} \nu' \quad \text{iff} \quad \text{there is } \nu'' \text{ such that } \nu R_{\pi_1} \nu'' R_{\pi_2} \nu' \]
  \[ \nu R_{\pi_1 \sqcup \pi_2} \nu' \quad \text{iff} \quad \nu R_{\pi_1} \nu' \text{ or } \nu R_{\pi_2} \nu' \]
  \[ \nu R_{\varphi?} \nu' \quad \text{iff} \quad \nu = \nu' \text{ and } \nu \models \varphi \]
Valid in introspective valuations

- S5 axiom schemas valid for $K_i$:
  \[ K_i \varphi \rightarrow \varphi \]
  \[ K_i \varphi \rightarrow K_i K_i \varphi \]
  \[ \neg K_i \varphi \rightarrow K_i \neg K_i \varphi \]

- Fixed-point axiom schema valid for $CK$:
  \[ CK \varphi \leftrightarrow \varphi \land \bigwedge_i K_i \ CK \varphi \]

- Induction axiom schema invalid for $CK$:
  \[ \varphi \land CK (\varphi \rightarrow \bigwedge_i K_i \ CK \varphi) \not\rightarrow CK \varphi \]
Properties of DEL-PAO, ctd.

• sound and complete axiomatization
  1. reduction axioms for $K_i$, $CK$, $[\pi]$

\[
K_i \varphi \leftrightarrow [\pi_i, ATM(\varphi)] \varphi
\]

\[
CK \varphi \leftrightarrow [\pi_{Agt}, ATM(\varphi)] \varphi
\]

\[
[\pi \bigoplus \pi'] \varphi \leftrightarrow \ldots
\]

\[
\ldots
\]

\[
[+\alpha] \varphi \leftrightarrow \ldots
\]

\[
[-\alpha] \varphi \leftrightarrow \ldots
\]

2. introspection axioms:

\[
S_i S_i \alpha
\]

\[
JS JS \alpha
\]

\[
JS S_i S_i \alpha
\]

\[
JS \alpha \rightarrow S_i \alpha
\]

\[
JS \alpha \rightarrow JS S_i \alpha
\]

3. modus ponens

4. rules of equivalence for $K_i$, $CK$, $[\pi]$

Herzig
Properties of DEL-PAO, ctd.

- complexity: SAT and MC both \( \text{PSPACE-complete} \)
  1. MC can be polynomially reduced to SAT
  2. SAT can be polynomially reduced to MC
  3. lower bound for MC: polynomial encoding of QBF
     \[ \nu \models \forall p. \varphi \iff \nu \models [+p \cup -p] \varphi \]
  4. upper bound for MC: polynomial encoding into Dynamic Logic of Propositional Assignments DL-PA [HLTM11, BHT13]

\( \implies \) better than SAT for \( S5^\text{CK}_n \) (\( \text{EXPTIME-complete} \))
Application: the gossip problem
The gossip problem

[Baker&Shostak, Discrete Mathematics 1972]

- six friends each with a secret $\sigma_i$
- they can call each other to exchange every secret they know
- how many calls to spread all secrets among all friends?

(picture from [vDK15])
The gossip problem

- goal: shared knowledge

\[ \text{EK } \varphi = \bigwedge_{i \in \text{Agt}} K_i \varphi \]

(‘everybody knows’)

- optimal algorithm: 8 calls to obtain \( \text{EK}(\sigma_1 \land \cdots \land \sigma_6) \)
  - for \( n \) agents: \( 2(n-1) \) calls

- versatile:
  - reasoning about social networks, disease spreading, ...
    \[ \implies \text{take some network structure into account} \]
  - different kinds of protocols
    \[ \implies \text{distributed vs. centralized} \]

- hot topic in the DEL community:
  - [AvDGvdH14, vDK15]

- multiagent planning’s blocksworld?
The gossip problem in DEL-PAO

call = program:

\[
call_{ij} = ((K_i \sigma_1 ?; +S_j \sigma_1) \sqcup \neg K_i \sigma_1 ?); \ldots ; ((K_i \sigma_6 ?; +S_j \sigma_6) \sqcup \neg K_i \sigma_6 ?);
((K_j \sigma_1 ?; +S_i \sigma_1) \sqcup \neg K_j \sigma_1 ?); \ldots ; ((K_j \sigma_6 ?; +S_i \sigma_6) \sqcup \neg K_j \sigma_6 ?)
\]
The gossip problem in DEL-PAO

call = program:
\[
call_{ij} = ((K_i \sigma_1; +S_j \sigma_1) \sqcup \neg K_i \sigma_1); \cdots ; ((K_i \sigma_6; +S_j \sigma_6) \sqcup \neg K_i \sigma_6); \cdots ; ((K_j \sigma_1; +S_i \sigma_1) \sqcup \neg K_j \sigma_1); \cdots ; ((K_j \sigma_6; +S_i \sigma_6) \sqcup \neg K_j \sigma_6)
\]

For valuation \( \nu \) such that \( \sigma_i \in \nu \) and such that \( S_i \sigma_j \in \nu \) iff \( i=j \):

\[
\nu \models \left[ \text{call}_{12}; \text{call}_{34}; \text{call}_{56}; \text{call}_{13}; \text{call}_{45}; \text{call}_{16}; \text{call}_{24}; \text{call}_{35} \right] \text{EK} \left( \bigwedge_{1 \leq j \leq 6} \sigma_j \right)
\]

\[
\nu \models \langle ( \bigsqcup_{1 \leq i,j \leq 6} \neg S_i \sigma_j; \text{call}_{ij}) \rangle^6 \text{EK} \left( \bigwedge_{1 \leq j \leq 6} \sigma_j \right)
\]

\[
\nu \models \left[ ( \bigsqcup_{1 \leq i,j \leq 6} \neg S_i \sigma_j; \text{call}_{ij} ) \right]^5 \neg \text{EK} \left( \bigwedge_{1 \leq j \leq 6} \sigma_j \right)
\]
The gossip problem: attaining higher-order shared knowledge

- attain shared knowledge of level 2:
  \[ EK EK \left( \bigwedge_{1 \leq j \leq 6} \sigma_j \right) \]
- attain shared knowledge of level \( k \):
  \[ EK^k \left( \bigwedge_{1 \leq j \leq 6} \sigma_j \right) \]
- algorithm with \((k+1) \times (n-1)\) calls to attain shared knowledge of order 2 [Herzig & Maffre, submitted]
  - optimal?
Adding public announcements
Semantics: add current info state

[Herzig et al., ongoing]

- idea: evaluate epistemic formulas not only wrt agents’ observations, but also wrt the current information state [CS15]
  - current information state = set of valuations $\mathcal{W}$
  - pointed model = information state $\mathcal{W} +$ valuation $\nu$
- language: add public announcements
- truth conditions:
  \[
  \mathcal{W}, \nu \models [\psi!]\varphi \iff \mathcal{W}, \nu \models \psi \implies \|\psi\|_{\mathcal{W}}, \nu \models \varphi
  \]
  \[
  \mathcal{W}, \nu \models K_i \varphi \iff \mathcal{W}, \nu' \models \varphi \text{ for every } \nu' \in \mathcal{W} \text{ s.th. } \nu \sim_i \nu'
  \]
- properties:
  - reduction axioms $\implies$ decidable
  - PSPACE complete
Application: the muddy children puzzle
Higher-order observability  Gossip  Adding announcements  Muddy children  Boolean games

Application: the muddy children puzzle

for \( \nu \) such that \( S_i m_j \in \nu \) iff \( i \neq j \) and \( JS_i m_j \in \nu \) for all \( i,j \):

- ignorance persists for \( n-2 \) rounds

\[
\nu \models \text{Ignorance}
\]
\[
\nu \models \left[ \left( \bigvee_i m_i \right) ! \right] \text{Ignorance}
\]
\[
\nu \models \left[ \left( \bigvee_i m_i \right) ? ! \right] \text{Ignorance} ? ! \text{Ignorance}
\]
\[
\nu \models \left[ \left( \bigvee_i m_i \right) ? ! \right] \text{Ignorance} ? ! \text{Ignorance}^{n-2}
\]

- shared and even common knowledge comes after \( n-1 \) rounds

\[
\nu \models \left[ \left( \bigvee_i m_i \right) ? ! \right] \text{Ignorance} ? ! \text{Ignorance}^{n-1} \text{EK} \bigwedge_i m_i
\]
\[
\nu \models \left[ \left( \bigvee_i m_i \right) ? ! \right] \text{Ignorance} ? ! \text{Ignorance}^{n-1} \text{CK} \bigwedge_i m_i
\]

with \( \text{Ignorance} = \bigwedge_i (\neg K_i m_i \land \neg K_i \neg m_i) \)
<table>
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**Application: boolean games**
Putting things together: accounting for epistemic boolean games

- boolean games
  - exclusive and exhaustive propositional control:
    \[
    \left( \bigwedge_{i \neq j} \neg (C_i p \land C_j p) \right) \land \left( \bigvee_{i \in \text{Agt}} C_i p \right)
    \]
  - strategy of agent \(i\) = truth values of \(i\)'s variables
    \(\implies\) strategy profile = valuation
  - goal of agent \(i\) = propositional formula \(\gamma_i\)
    \(\implies\) utility of strategy profile \(v\) is 1 if \(v \models \gamma_i\); is 0 otherwise
  - strategy profile \(v\) is a Nash equilibrium iff
    \[
    v \models \bigwedge_{i \in \text{Agt}} (\Diamond_i \gamma_i \rightarrow \gamma_i)
    \]
Putting things together: accounting for epistemic boolean games

- epistemic boolean games:
  - generalize propositional variables to atoms: $S_i C_j p$, ...  
  - generalize goals to epistemic formulas
  - same definitions: strategy, Nash equilibrium, ...

example:
- agent 1 has a secret, $s_1$, and 2 has a secret, $s_2$
- agent $i$ may privately communicate his secret to $j$: $+S_j s_i$
- both have goal of ‘fair division of information’:

$$\gamma_1 = \gamma_2 = K_1 s_2 \leftrightarrow K_2 s_1$$

example:
- ...and agent 3 shouldn’t learn anything:

$$\gamma_1 = \gamma_2 = (K_1 s_2 \leftrightarrow K_2 s_1) \land \neg K_3 s_1 \land \neg K_3 \neg s_1 \land \neg K_3 s_2 \land \neg K_3 \neg s_2$$
Conclusion and future work

- \( \text{DEL-PAO} = \) dynamic epistemic logic based on visibility
  - higher-order observations
    - no common knowledge of who sees what
- add public announcements
  - information state
- add propositional control: \( \text{DEL-PAOC} \)
- interesting complexity
- future work:
  - ?? from knowledge to belief
    - problem: guarantee introspection
Maduka Attamah, Hans van Ditmarsch, Davide Grossi, and Wiebe van der Hoek.

Knowledge and gossip.


Philippe Balbiani, Andreas Herzig, and Nicolas Troquard.

Dynamic logic of propositional assignments: a well-behaved variant of PDL.


Tristan Charrier and Francois Schwarzentruber.

Mental programs and arbitrary announcements.


Andreas Herzig, Emiliano Lorini, and Faustine Maffre.

A poor man’s epistemic logic based on propositional assignment and higher-order observation.

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A dynamic logic of normative systems.

Wiebe van der Hoek, Petar Iliev, and Michael Wooldridge.
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Knowledge and control.

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*Epistemic Puzzles*.
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