The Black Top Hat function applied to a DEM: A tool to estimate recent incision in a mountainous watershed (Estibère Watershed, Central Pyrenees)

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[1] The Top Hat Transform function is a grey-level image analysis tool that allows extracting peaks and valleys in a non-uniform background. This function can be applied onto a grey-level Digital Elevation Model (DEM). It is herein applied to quantify the volume of recent incised material in a mountainous Pyrenean watershed. Grey-level Closing operation applied to the Present-Day DEM gives a new image called "paleo" DEM. The Black Top Hat function consists in the subtraction of the "paleo" DEM with the Present-Day DEM. It gives a new DEM representing all valleys whose sizes range between the size of the structuring element and the null value as no threshold is used. The calculation of the incised volume is directly derived from the subtraction between the two DEM's. The geological significance of the quantitative results is discussed. *INDEX TERMS:* 1815 Hydrology: Erosion and sedimentation; 1824 Hydrology: Geomorphology (1625); 3299 Mathematical Geophysics: General or miscellaneous

1. Introduction

[2] The Top Hat Transform function is a mathematical morphology function that allows peaks and valleys extraction in a 1D signal and 2D image. It is considered as an adaptive threshold because the resulting image is an image without local background levels. This function is particularly used in order to extract light intensity of peaks in a fluorescence or luminescence biology sample [*Maire et al.*, 2000; *Meyer*, 1979]. It can also be used to suppress halo or shading in the case of images acquisition. In geomorphology, this function is still scarcely used. For example, it was used to help in understanding the sedimentary dynamics of an estuary basin. The Top Hat Transform was applied to a DEM elaborated from a topographic map in order to determine the different relief patterns [*Brouchoud et al.*, 1993].

[3] This paper shows the application of The Top Hat Transform method in to a high-precision DEM as a relevant tool for estimating incised material corresponding to recent fluviatil erosion processes in a Pyrenean watershed.

2. Top Hat Transform Formalism

[4] The efficiency of The Top Hat Transform was introduced by Meyer in 1979 for Cytology applications [*Meyer*, 1979]. The mathematical formulation is based on set concepts [*Coster and J.L.*, 1989; *Serra*, 1982; 1988].

[5] When considering the elementary grey-tone mathematical morphology operations: let f(x) represent the numerical values in grey levels in a pixel located in x and u the considered point. The transformation of the image is named f(X) wherein X is the corresponding set of all pixels of the image. With the help of a structuring element λ centred on x, the following transformations are defined:

$$\text{Erosion}: E^{\lambda}f(X) = \inf\{f(u) : u \in \lambda_x\}$$
$$\text{Dilation}: D^{\lambda}f(X) = \sup\{f(u) : u \in \lambda_x\}$$

Thus, the following functions are then defined:

$$\text{Opening}: O^{\lambda}(X) = D^{\lambda}\left(E^{\lambda}f(X)\right)$$
$$\text{Closing}: C^{\lambda}(X) = E^{\lambda}\left(D^{\lambda}f(X)\right)$$

[6] The Opening function is commonly used to separate clear zones and the Closing function to join them. In the same way, dark zones can be joined with Opening operations and clear zones with Closing operations. The Opening and Closing functions are the basis of The Top Hat Transform function. It consists of an extraction of peaks and valleys with a size condition corresponding to the size of the considered structuring element [*Meyer*, 1979].

[7] When considering The White Top Hat (WTH) mathematical function, let t = a level of threshold. The peaks extraction in a grey levels image is defined as follows:

$$\text{WTH} = \left\{x : f(x) - O^{\lambda}(X) \geq t\right\}$$

The valley's extraction in a grey levels image is done by The Black Top Hat (BTH) function defined as follows:

$$\text{BTH} = \left\{x : C^{\lambda}(X) - f(x) \geq t\right\}$$

[8] The threshold value (t) permits to extract specifically the peaks and valleys without extracting the artefactual values that correspond to the weakest grey-levels. Hence, only the highest peaks or deepest valleys are retained. Both functions permit the extraction of peaks and valleys based on width and height criterions as shown on Figures 1a and 1b.

3. Outlines on Geological and Geomorphologic Settings

[9] The Estibère Watershed belongs to the Néouvielle Granitic Massif located in the central part of the Pyrenean Range and belongs to the upper part of the Garona Basin (Figure 2). This watershed is entirely composed of a late Hercynian granitic massif. [*Gleizes et al.*, 2001; *Lamouroux et al.*, 1980]. Geomorphologically speaking, the Estibère Watershed coincides with an inherited Würmian (80,000 to 10,000 yrs B.P.) glacial valley incised by a number of small fluvial valleys considered to have originated in the Late Würmian Ice Melting Phase. These fluvial valleys continue to this day to be formed by the incision processes (Figure 2). The

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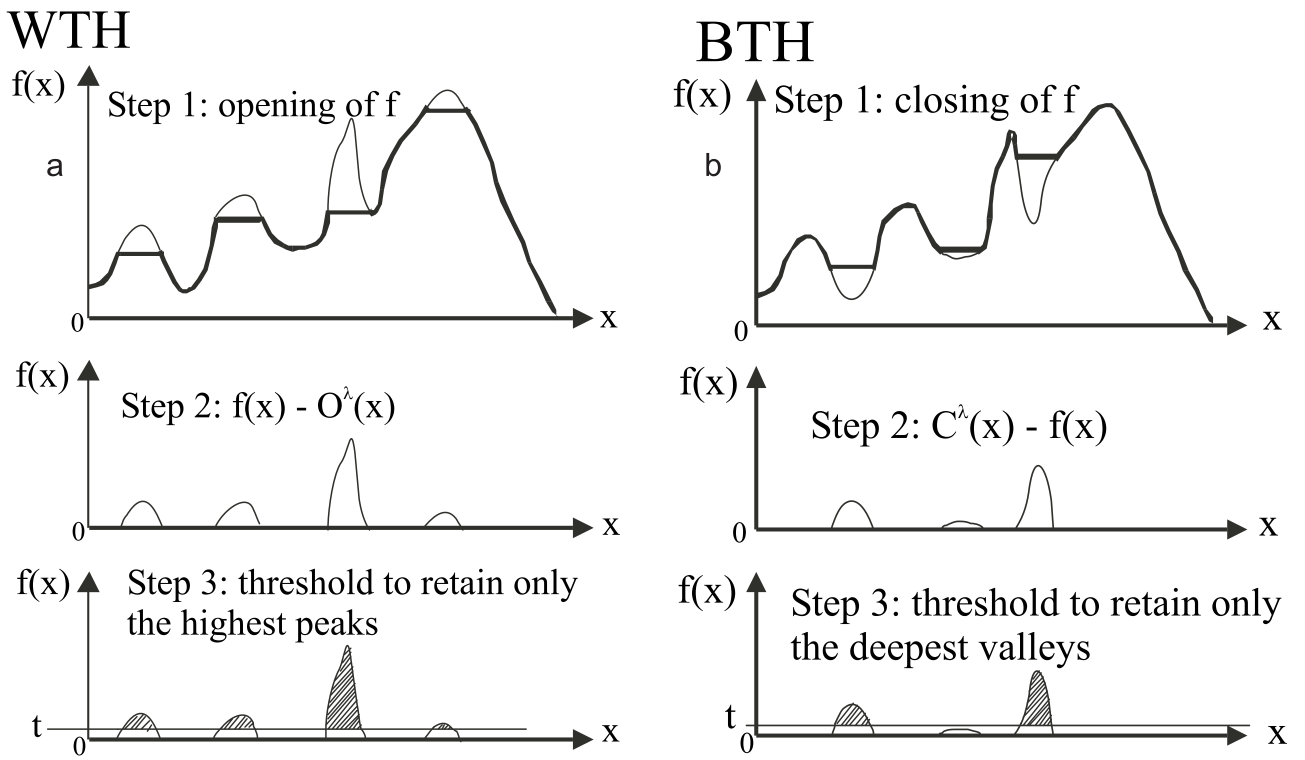

Figure 1. (a) 1D profiles of highest peaks after extraction by The White Top Hat function (t = threshold value). (b) 1D profiles of valleys after extraction by The Black Top Hat function (t = threshold value).

Estibère Watershed shows flat valleys with several lakes and peat bogs at the base of glacial depressions. Terraces of same altitude ("T" on Figure 2) adjacent to small Holocene s.l. (i.e. latest Würmian to Present-Day) valleys ("i" on Figure 2) marking the base of the latest widespread glacial episode which took place immediately before the Late Würmian Ice Melting Phase. The beginning of this period has been dated in several sites in the Pyrenees to around 14,000 years B.P. e.g. [*Jalut et al.*, 1988].

4. Objective

[10] The study of transversal profiles in the Present-Day DEM leads to the identification of Late Würmian terraces located at the same altitude and adjacent to Holocene (s.l) valleys. One example of these terraces in the Estibère Watershed is seen in (Figure 2). The objective is to estimate the incised volume in between the beginning of the Latest Ice-Melting Phase and Present-Day. In order to achieve this objective it is assumed that the reconstructed "paleo" DEM represents the surface of the valley at the time of the Late Würmian Ice Melting Phase.

5. Methodology and Results

[11] A high-precision DEM (10 m accuracy) built from a digitised topographic map (Figure 2, bottom) allowed for the edification of longitudinal profiles. These crossections show stair morphology marked by slight depressions and ledges (Figure 3). Moreover, the last glacial erosion surface shows a typical U profile cut by Holocene (s.l.) Valleys (Figure 3).

[12] A new DEM of the interest zone, called "paleo" DEM, has been built by filling the Holocene Valleys from the Present-Day DEM.

[13] A Closing operation applied to the Present-Day DEM fills the Holocene Valleys s.l. (i) in between the Late Würmian terraces (T) and reconstructs a "paleo" DEM. This is a representation of the topography of the glacial basin prior to the ice melting phase (Figure 3). It is postulated that the glacial Würmian erosion has entirely levelled Pre-Würmian landforms specifically those formed by incision processes. The subtraction of the Present-Day DEM from the "paleo" DEM gives a new DEM, which is an image of the supposed incised volume by fluvial Holocene (s.l.) processes (Figure 4).

[14] The Black Top Hat (BTH) function consists of the subtraction of the two DEMs, but without performing a threshold and taking into account all the minor valleys. The size of the structuring element, which the BTH function was applied to, corresponds to the greatest distance between terraces, estimated around 300 meters (Figure 3). Consequently, the BTH operation was computed with a circular structuring element corresponding to 150 m of radius. The isotropic characteristic of the structuring element allows to take into account all directions of the valleys. Calculation of the incised volume is derived directly from the difference between the two DEMs. The quantitative results are as follows:

Incised volume equals to 54. 10^6 m^3 ± 6.10^6 m^3
Estibère basin area equals to 55.10^7 m^2 ± 1.10^7 m^2

[15] Calculation of the incision rate from the Late Würmian Ice-Melting Phase to Present-Day assumes that the mean density of the granitic rocks equals 2.7 gr/cm^3 e.g. [*Olivier et al.*, 1999]. In this case, the mean incision rate ≈1500t/km^2/yr in the last 14,000 years. This rate is about 60 times greater than the present day mechanical erosion rate estimated at 28t/km^2/yr for the upper Garona Basin [*Probst*, 1992]. It is, however, 2.5 times greater than the mechanical erosion for the Bramapoutre Himalayan Watershed (estimated at 640t/km^2/yr [*Wolfgang and Probst*, 1998]) that is also mainly composed of granitic and metamorphic rocks. Thus, when comparing the two given watershed erosion rates, the results are unrealistic in the Estibère Watershed.

[16] The question is how to interpret this abnormally high rate. As mentioned above, the rate was calculated by taking into account

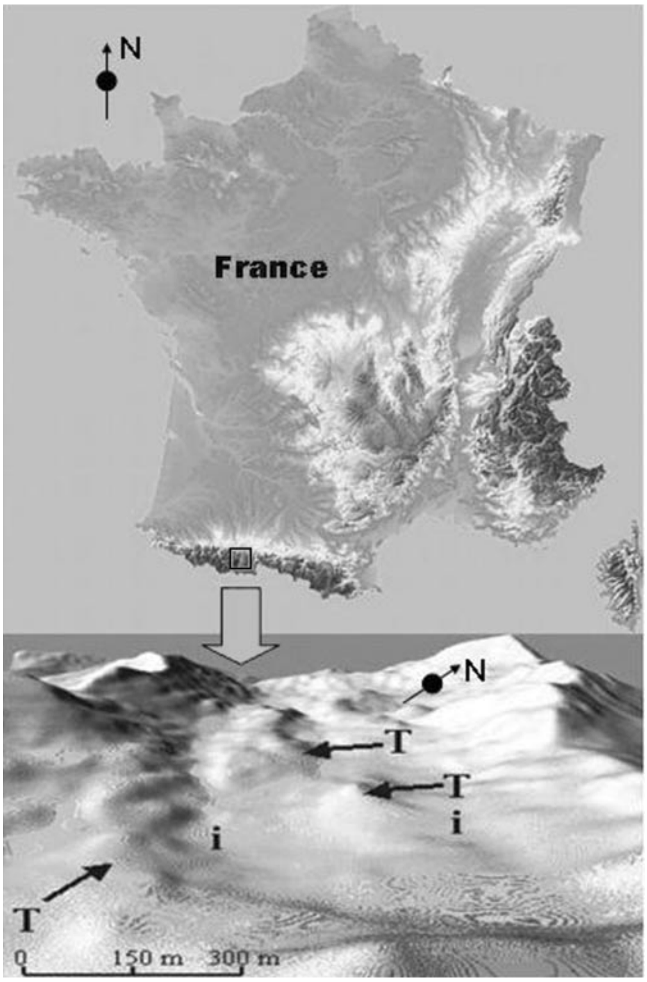

Figure 2. (Top) Localisation of the Estibère basin. (Bottom) DEM of the Estibère basin (T: late Würmian terraces, i: Holocene (s.l.) fluviatil incisions).

that all the Ante-Würmian incisions were entirely levelled by the Würmian glacial erosion. Some observations are in disagreement with this hypothesis. One of these observations concerns the fact that the granitic rocks forming the basement of the Estibère watershed are highly cohesive rocks which are not easily mechanically eroded. Another suggests that the glacial erosional stress was

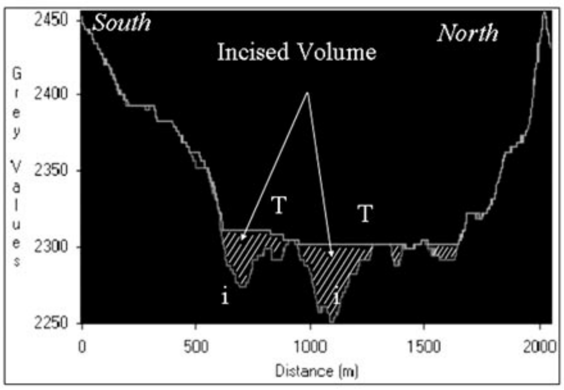

Figure 3. Comparison of Present-Day DEM line profile (dark grey) with the "paleo" (latest Würmian) DEM (grey) (T: late Würmian terraces, i: supposed Holocene (s.l.) fluviatil incisions).

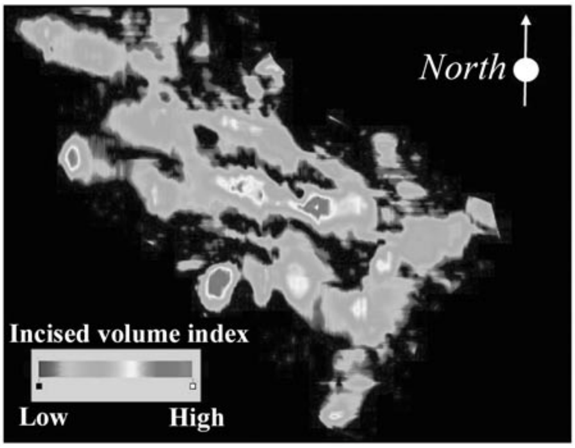

Figure 4. Pseudo-colour image (represented in grey tones) of the DEM obtained by the Black Top Hat function without thresholding. It shows incised volume due to Holocene (s.l.) fluvial processes in the Estibère watershed.

not high enough due to the low degree of the slope (≈ 15° in average) on which the Estibère glacier was established. Also the evolution of the climate of the Late Würmian period from wet to dry temperate conditions did not trigger the mechanical erosion processes [*Jalut et al.*, 1988].

[17] Thus, it is more likely to consider that the Ante-Würmian fluvial incisions were not entirely levelled during the Würmian Glacial Phase and that the calculated volume corresponds to a combination of Ante-Würmian (at least Post-Rissian?) and Post-Würmian incised volumes. Consequently, the high incision calculated rate does not correspond to the Late Würmian to Present-Day incision rate. For a more precise estimation of the latter there are two possibilities: (i) to find markers of the holocene incision within the valleys by analysing cosmogenic exposures using radionucleides (^{10}Be, ^{26}Al) as in the Himalayan Range [*Sharma et al.*, 1998] (ii) to calculate the incised volume using the BTH method within lower parts of the Pyrenean glacial valleys where the antecedent incisions should be completely levelled during the Würmian Glacial Stage as more erodible materials prevail (work in progress).

6. Conclusion

[18] By applying The Black Top Hat Transform function to a high-precision DEM, we have developed a method for estimating the recent incision rate produced by the fluviatil processes in an inherited glacial Pyrenean watershed. The Black Top Hat function allows extraction of the valleys of a given size depending upon the size of the chosen structural element. Further, without applying the threshold operation permits to take into account all valleys. This leads to a more precise estimation of the total incised volume. Note that the BTH function is an adaptive filter independent from local altitudes. It allows filling up valleys whose lengths are equal or inferior to the size of the structuring element independently of their relative topographic location.

[19] Accuracy of the estimation of the erosion rate depends on two main parameters: (i) determination of the size of the structuring element which herein corresponds to the mean distance between each terrace adjacent to the assumed Holocene (s.l.) incisions, (ii) accuracy of determination of area of the watershed affected by processes of glacial erosion. The accuracy in these two estimations produces the error rate of ±11%. Note that this rate is dependant solely upon the precision of the radius of the structuring element chosen for BTH implementation. The flattened shape of late Würmian terraces induces several pixels of uncertainty when

choosing the radius size. This method that allied the efficiency of the BTH function and the high morphological precision of a DEM, can be applied to other cases of analysis of mechanical erosion. It requires knowledge of the dating of the successive incision phases in order to reconstruct the "paleo" DEM more precisely.

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References

Brouchoud, H., H. Regnaud, and P. Gouery, Morphologie mathématique appliquée à un site d'estuaire An Estuary site analysed by mathematical morphology processes. Bull. Inst. Géol. Bassin d'Aquitaine, 53, 209–219, 1993.

Coster, M. and J. L. C., Précis d'analyse d'images, CNRS, 1989.

Gleizes, G., D. Leblanc, P. Olivier, and J. L. Bouchez, Strain partitioning in a pluton during emplacement in transpressional regime: the example of the Néouvielle granite (Pyrenees), Geol. Runds in press, 2001.

Jalut, G., V. Andrieu, G. Delibrias, and M. Fontugne, Paleoenvironment of the valley of Ossau (western French Pyrenees) during the last 27000 years, Pollen et Spores, 3–4, 357–394, 1988.

Lamouroux, C., J. C. Soula, J. Deramond, and P. Debat, Shear zones in the granodioritic massifs of the Central Pyrenees and the behaviour of these massifs during the Alpine Orogenesis, *J. Struct. Geol.*, 2, 49–53, 1980.

Maire, E., Development of an ultra-low light level luminescence image analysis system for dynamic measurements of transcriptional activity in living and migrating cells, *Analytical Biochemistry*, *280*(1), 118–127, 2000.

Meyer, F., Cytologie quantitative et morphologie mathématique. Thèse de docteur-ingénieur Thesis, Ecole des Mines, Paris, 1979.

Olivier, P., P. Ameglio, H. Richen, and F. Vadeboin, Emplacement of the Aya Variscan granitic pluton (Basque Pyrenees) in a dextral transcurrement regime inferred from a combined magneto-structural and gravimetric study, *Journal of the Geological Society, London*, *156*, 991–1002, 1999.

Probst, J. L., Géochimie et Hydrologie de l'érosion continentale. Mécanismes, bilan global actuel et fluctuations au cours des 500 derniers millions d'années, Science Géologique, 94, 161, 1992.

Serra, J., Image Analysis and Mathematical Morphology, Theoretical Advances, Academic Press, London, 422 pp., 1982.

Serra, J., Image Analysis and Mathematical Morphology, 2. Academic Press, London, 411 pp., 1988.

Sharma, K. K., Z. Y. Gu, D. Lal, M. W. Caffee, and J. Southon, Late quaternary morphotectonic evolution of Upper Indus valley profile: a cosmogenic radionucleide study of river polished surfaces, *Current Science*, *75*(4), 336–371, 1998.

Wolfgang, L., and J. L. Probst, River sediment discharge to the oceans: Present-Day controls and global budgets, *American Journal of Science*, *298*(APRIL), 265–295, 1998.

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