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Comparison of simulation models in terms of quantity and allocation of land change

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1. Introduction

Land Change Models can be useful tools for environmental and geomatics research concerning land use and cover change (LUCC) (Paegelow et al., 2013). The simulation maps obtained from LUCC models help us to understand, forecast and anticipate the future evolution for a variety of applied environmental problems. One of the most important challenges is to clarify the validity of the outputs from the models.

A study area’s rate and pattern of LUCC can influence a simulation’s accuracy. If the real transitions among land categories constitute a small portion of the study area, then it can be difficult to predict the changes accurately, especially when the data have errors (Pontius et al., 2008; Pontius and Petrova, 2010). Furthermore, if the changes during the calibration interval are not stationary with the changes during the validation interval, then an extrapolation from the calibration interval to the validation interval will probably have systematic errors (Pontius and Neeti, 2010). Moreover, land changes involve complex processes that are shaped by dynamic, non-linear human-nature interactions, which can be difficult for available variables and algorithms to capture (Pérez-Vega et al., 2012; Kolb et al., 2013).

The choice of the model is a subject of debate concerning the validation of models (Villa et al., 2007; Paegelow and Camacho Olmedo, 2008; Pontius et al., 2008; Fuller et al., 2011; Mas et al., 2011; Wang and Mountrakis, 2011; Sinha and Kumar, 2013). Some research attempts to test whether one model is more accurate than another model. However, a single model can create various outcomes depending on how the user chooses the model’s parameters. If the choice of the parameters within a single model produces greater variation in accuracy than the choices among alternative models, then it makes little sense to report that one model is more accurate than another model (Pontius and Malanson, 2005). Model comparison can be challenging, because each model offers a variety of choices at each stage in a modeling application.

Our article concerns how to compare the outputs from models that simulate transitions among categories over time. We illustrate the concepts by comparing two land change models: Land Change Modeler and Cellular Automata Markov. We show how the modeling options influence the quantity and allocation of simulated transitions, and how to compare output maps from pairs of model runs with respect to a reference map of transitions during the validation interval. We recommend that the first step is to assess the quantity of each transition and to determine the cause of the variation in quantity among model runs. The second step is to assess the allocation of transitions and to determine the cause of the variation in allocation among model runs. The separation of quantity and allocation of the transitions is a helpful approach to communicate how models work and to describe pattern validation.
with other models, see Paegelow and Camacho Olmedo (2008) and Mas et al. (2014). For applications of LCM, see Aguejdad and Houet (2008), Dang Khoi and Murayama (2010), Fuller et al. (2011) and Sangermano et al. (2012). For applications of CA_MARKOV, see Paegelow and Camacho Olmedo (2005), Shirley and Battaglia (2008), Kamusoko et al. (2009), Mobaied et al. (2011), Behera et al. (2012), Memarian et al. (2012) and Nejadi et al. (2012). Our goals are to show: (1) how the modeling options influence the quantity and allocation of simulated changes, and (2) how to compare output maps from pairs of model runs with respect to a reference map of change during the validation interval.

2. Methods

2.1. Data

The study area is in Spain’s Segura River Basin, which has undergone profound territorial and economic transformations in recent decades. The primary change by far has been the transition from rainfed crops to irrigated crops. This transition is driven by the development of water-related infrastructures and the increase in the water supply (Gómez and Grindlay, 2008; Gómez Espín et al., 2011). A secondary change has been urban growth, which is driven by the development of transportation and communication infrastructures.

Fig. 1 shows the specific study area, which consists of 2300 square kilometers in the Murcia province. The maps of land use and cover have four categories from the Corine Land Cover dataset: 1 Urban, industrial and transport uses; 2 Natural vegetation, unproductive land and water; 3 Irrigated crops; 4 Rainfed crops. The remainder of this article refers to these categories as: Urban, Natural, Irrigated, and Rainfed. Corine maps at 1990 and 2000 are used for model calibration. The calibration, unproductive land and water; 3 Irrigated crops; 4 Rainfed crops. The Corine Land Cover dataset: 1 Urban, industrial and transport uses; 2 Natural vegetation, unproductive land and water; 3 Irrigated crops; 4 Rainfed crops. The remainder of this article refers to these categories as: Urban, Natural, Irrigated, and Rainfed. Corine maps at 1990 and 2000 are used for model calibration. The calibration, unproductive land and water; 3 Irrigated crops; 4 Rainfed crops. The Corine Land Cover dataset: 1 Urban, industrial and transport uses; 2 Natural vegetation, unproductive land and water; 3 Irrigated crops; 4 Rainfed crops. The

2.2. Components of models

2.2.1. Quantity of change

LCM and CA_MARKOV use a Markov matrix to extrapolate the quantity of each simulated transition. The MARKOV module in the Idrisi software computes a Markov matrix based on the maps at the two time points that define the calibration interval. The rows of the matrix show the host categories at the time point t0 and the columns show the claimant categories at the time point t1. The Markov matrix records the proportion of each host category at time t0 that transitions to a claimant category at time t1. Persistence of each category is shown in a diagonal entry of the matrix. Changes are in the off-diagonal entries of the matrix. The calibrated matrix is then used to extrapolate the quantity of each simulated transition beyond time t1. If the duration of the calibration interval is equal to the duration of the validation interval, then the application of the Markov matrix is a straightforward application of matrix multiplication.

For our case study, the calibration interval is ten years from t0 = 1990 to t1 = 2000, while the validation interval is six years from t1 = 2000 to t2 = 2006. Table 2 shows how Idrisi accounts for this difference in durations. For each Markov entry, Idrisi interpolates a curve from t1 to t2 (t1−t0), which means in our case from 2000 to 2010. The interpolated curve allows Idrisi to compute the proportion at any year between t1 and t2 + (t1−t0). For example, Table 2 shows that the proportion of Rainfed transition to Irrigated during 2000−2010 is 0.2525, which is set to be identical to the proportion during the decade of the calibration interval. The interpolated curve is then used to compute the proportion at any year between 2000 and 2010. For example, the proportion at the year 2006 is 0.1696. A more detailed description of the MARKOV module is in the Idrisi Selva Help System, Mas et al. (2011, 2014) and Sinha and Kumar (2013).

2.2.2. Allocation of change

The Markov extrapolation specifies the target number of pixels for transition during the validation interval, then LCM and CA_MARKOV allocate those transitions spatially within the study area. To accomplish the allocation, both models use maps that specify the priority for each pixel to experience gain of a particular claimant category. LCM uses a transition potential map for each group of transitions in order to allocate the simulated transitions. Table 3 shows how we placed the transitions into four groups. For example, the first group is called urban gain, which contains three transitions to Urban. Table 3 shows seven transitions of the possible twelve transitions among four categories. We ignore five small transitions. Specifically, we assume that Urban never loses and that Irrigated never transitions to Natural or Rainfed. LCM uses a multi-layer perceptron (MLP) to create a transition potential map for each transition group (Eastman et al., 2005). The MLP reads maps of independent variables and maps of the transitions during the calibration interval to produce a map of transition potential for each group. Our independent variables are topographic variables, protected areas, distance to irrigation canals, and accessibility to roads and human settlements. CA_MARKOV uses a suitability map for each claimant category in order to allocate the simulated transitions. We used multi-criteria evaluation (MCE) and the same independent variables to create one suitability map for each of the four categories (Eastman et al., 1995).
that show three types of information.

CA_MARKOV without maps in columns. Horizontal lines separate the four sub-models in LCM.

Table 1

<table>
<thead>
<tr>
<th>Category</th>
<th>Gains</th>
<th>Losses</th>
<th>Total changes</th>
<th>Swap</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urban</td>
<td>2.01</td>
<td>1.32</td>
<td>2.03 1.32</td>
<td>0.04</td>
<td>1.99</td>
</tr>
<tr>
<td>Natural</td>
<td>0.30</td>
<td>0.39</td>
<td>1.01 0.68</td>
<td>1.07</td>
<td>0.78</td>
</tr>
<tr>
<td>Irrigated</td>
<td>11.67</td>
<td>11.47</td>
<td>1.35 0.80</td>
<td>12.27</td>
<td>1.60</td>
</tr>
<tr>
<td>Rainfed</td>
<td>0.99</td>
<td>0.25</td>
<td>12.59 11.95</td>
<td>12.20</td>
<td>1.50</td>
</tr>
<tr>
<td>Total</td>
<td>14.97</td>
<td>13.43</td>
<td>14.97 13.43</td>
<td>2.66</td>
<td>1.44</td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th>Year</th>
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</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>0.0000</td>
</tr>
<tr>
<td>2001</td>
<td>0.0529</td>
</tr>
<tr>
<td>2002</td>
<td>0.0774</td>
</tr>
<tr>
<td>2003</td>
<td>0.1013</td>
</tr>
<tr>
<td>2004</td>
<td>0.1247</td>
</tr>
<tr>
<td>2005</td>
<td>0.1475</td>
</tr>
<tr>
<td>2006</td>
<td>0.1696</td>
</tr>
<tr>
<td>2007</td>
<td>0.1912</td>
</tr>
<tr>
<td>2008</td>
<td>0.2122</td>
</tr>
<tr>
<td>2009</td>
<td>0.2326</td>
</tr>
<tr>
<td>2010</td>
<td>0.2525</td>
</tr>
</tbody>
</table>

Table 3

<table>
<thead>
<tr>
<th>Transitions</th>
<th>Urban</th>
<th>Natural</th>
<th>Irrigated</th>
<th>Rainfed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural to Urban</td>
<td>urban gain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Irrigated to Urban</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rainfed to Natural</td>
<td></td>
<td>natural gain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Natural to Irrigated</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rainfed to Irrigated</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Natural to Rainfed</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Results

3.1 Quantity of change

First, we overlay the map of Irrigated at 2000 with the Corine and simulated maps of Irrigated at 2006. These overlays show the spatial relationship between Irrigated at 2000 and the gain of Irrigated during 2000–2006.

Second, we overlay the Corine map of 2000, the Corine map of 2006, and a simulation map of 2006. This produces four types of pixels: misses, hits, false alarms and correct rejections. Miss means the Corine maps show change but the simulation shows persistence. Hit means the Corine maps show change and the simulation shows change. False alarm means the Corine maps show persistence but the simulation shows change. Correct rejection means the Corine maps show persistence and the simulation shows persistence.

Third, we compare the change shown by the Corine maps to the change simulated by pairs of models. We overlay three maps that show change versus non-change during the validation interval. The three maps are from: the Corine data, the simulation from model 1 and the simulation from model 2. Three maps of change versus non-change lead to eight possible combinations for each pixel. These eight possible combinations are expressed as two sets of maps. The first set shows four combinations for the Corine persistence areas. The second set shows four combinations for the Corine change areas. The definitions for the legend in the Corine persistence areas are as follows. DOUBLE CORRECT REJECTION means both models simulated persistence. FALSE ALARMS/CORRECT REJECTION means the first model simulated change and the second model simulated persistence. CORRECT REJECTION/FALSE ALARMS means the first model simulated persistence and the second model simulated change. DOUBLE FALSE ALARMS means both models simulated change. The definitions for the legend in the Corine change areas are as follows. DOUBLE HITS means both models simulated change. MISSES/HITS means the first model simulated persistence and the second model simulated change. HITS/ MISSES means the first model simulated change and the second model simulated persistence. DOUBLE MISSES means both models simulated persistence.

Some pixels have the possibility to transition to more than one claimant category. For example, a pixel of Natural can transition to any of the other three categories during the simulation. IDRISI's Multi-Objective Land Allocation (MOLA) algorithm selects the single claimant category for each pixel. MOLA determines the claimant category based on a minimum-distance-to-ideal-point rule and weighted ranks (Eastman et al., 1995; Mas et al., 2014).

CA_MARKOV uses temporal iterations, which the user must specify. We chose 6 iterations because 6 is the number of years during the simulation interval. MOLA runs once per iteration for CA_MARKOV. Therefore, MOLA ran six times for our application of CA_MARKOV.

CA_MARKOV allows for options concerning how to use its spatial filter. We ran CA_MARKOV in two ways with respect to the filter. The first run includes a spatial filter and the second run does not. We call the run with the filter “CA_MARKOV”, and the run without the filter “CA_MARKOV without filter”. Specifically, the first run of CA_MARKOV uses a 5 x 5 spatial filter, with rows (00100; 01110; 11111; 01110; 00100). The consequence of the filter is that the simulated gain of a claimant category tends to be near where the claimant category already exists. The second run of CA_MARKOV does have the effect of a spatial filter, meaning the filter has rows (000; 010; 000). The consequence is that CA_MARKOV without filter does not consider spatial proximity to an existing category when simulating the gain of that category.

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2.3 Validation via map overlay

We perform pattern validation using map overlays. The overlays produce maps that show three types of information.

Fig. 2 shows the percent loss of each host category according to the reference Corine data during the calibration and validation intervals and according to the Markov extrapolation during validation interval. The calibration interval has more years than the validation interval. Therefore, the reference losses during the calibration interval are greater than the extrapolation losses during the validation interval. Reference losses from Natural and Rainfed during the validation interval are greater than the corresponding extrapolated losses, which indicates that losses from Natural and Rainfed accelerated from the calibration interval to the validation interval. Meanwhile losses from Urban and Irrigated apparently decelerated from the calibration interval to the validation interval.

The output maps from LCM and CA_MARKOV have various amounts of change, in spite of the fact that both LCM and CA_MARKOV use a Markov matrix to extrapolate the number of pixels for each transition. One reason for the discrepancies is that the user can choose to ignore some transitions by setting those transitions to zero in the Markov matrix. Table 4 shows how a Markov matrix that includes all transitions extrapolates more change than a Markov matrix that has only seven transitions. Table 4 also shows that the output map from LCM matches the quantity of change from the Markov extrapolation with seven transitions. However, the output maps from both CA_MARKOV runs show less change than from the Markov extrapolations.

Table 5 reveals the details of the changes by showing the quantity of each transition for various matrices. The Markov
extrapolation for the seven modeled transitions matches the output from LCM, but the outputs from the CA_MARKOV runs do not. Both CA_MARKOV runs produce maps that have identical areas by category at 2006, and both CA_MARKOV runs show positive values for ten transitions. However, the sizes of the transitions differ between CA_MARKOV and CA_MARKOV without filter.

### 3.2. Allocation of change

Fig. 3 shows the spatial relationship between the Irrigated at 2000 and the gain of Irrigated during 2000–2006. The Corine data show that the gain of Irrigated occurs mostly in large patches, especially in the northwest. The CA_MARKOV output shows how the spatial filter causes the simulated gain of Irrigated to occur adjacent to the Irrigated at 2000. CA_MARKOV without filter and LCM have the ability to simulate the gain of Irrigated far from the Irrigated at 2000.

Fig. 4 shows the misses, hits, false alarms and correct rejections for the three simulations. The union of misses and hits is the change for all three simulations. The Figure of Merit is a measure that examines how the simulated change overlaps with the reference change, where 0% means no overlap and 100% means perfect overlap. Specifically, the Figure of Merit is the ratio of hits to the sum of misses, hits and false alarms (Pontius et al., 2008). The Figure of Merit is 10.4% for LCM, 10.5% for CA_MARKOV, and 10.2% for CA_MARKOV without filter.

Fig. 5 presents overlays of the three simulation pairs: LCM versus CA_MARKOV, LCM versus CA_MARKOV without filter, and CA_MARKOV versus CA_MARKOV without filter. Fig. 5a shows the results for the Corine persistence areas during 2000–2006, where a pixel of simulated persistence is a correct rejection and a pixel of simulated change is a false alarm. Fig. 5b shows the results for the Corine change areas during 2000–2006, where a pixel of simulated change is a hit and a pixel of simulated persistence is a miss. Fig. 5 includes one bar for each map to clarify the relative percent of each of the four combinations within each map.

The bars in Fig. 5a show that LCM versus CA_MARKOV without filter has the largest double correct rejections and double false alarms. This implies that LCM and CA_MARKOV without filter form the pair that is most similar to each other, in the region where the Corine data show persistence. The bars in Fig. 5b show that LCM versus CA_MARKOV without filter has the largest double hits and misses. This implies that LCM and CA_MARKOV without filter form
the pair that is most similar to each other, in the region where the Corine data show change.

The percent error at 2006 of the three model runs are all within two percentage points of each other, for both the Corine persistence areas and the Corine change areas. Consequently, if the Corine maps have more than two percentage points of error, then the models are indistinguishable in terms of predictive accuracy.

4. Discussion

4.1. Quantity of change

LCM and CA_MARKOV both use a Markov matrix to extrapolate the quantity of each transition and persistence. The Markov method of extrapolation is used also in other land change models, such as DINAMICA (Soares-Filho et al., 2002). The fact that the Markov matrix is integrated into various models has led to its generalized use within the community of land change modelers. However, the Markov matrix can experience conceptual problems when the calibration interval and the extrapolation interval have different durations. It would be convenient to compute a Markov matrix that extrapolates over an annual time step, regardless of the number of years of the calibration interval. However, Takada et al. (2010) showed that attempts to compute an annual Markov matrix can give multiple solutions or no solutions that have entries between 0 and 1. Flamenco-Sandoval et al. (2007) experienced one of these difficulties when trying to compute annual transition matrices, finding that negative transition proportions can appear in the annualized matrix. Idrisi software uses interpolation for the entries in the Markov matrix in order to compute a Markov entry between 0 and 1 for the desired extrapolation year (Clark Labs, 2006, 2010; Eastman, 2009).

All three simulations in our case study show less change than the reference change during the validation interval. One reason is that we chose to simulate only seven of the twelve possible transitions among the four categories. A second reason is that the two

Fig. 3. Each map shows Irrigated at 2000, Gain of Irrigated during 2000–2006, and Other for (a) LCM, (b) CA_MARKOV, (c) CA_MARKOV without filter, and (d) Corine.
CA_MARKOV output maps show less change than the amount of change extrapolated by the Markov matrix with seven transitions. However, the most obvious reason is the non-stationarity of the amount of change between the calibration interval and the validation interval. For our case study, the Corine data show acceleration of change from the calibration interval to the validation interval. This is most evident for the transition from Rainfed crops to Irrigated (Table 5).

4.2. Allocation of change

The use of the spatial filter in CA_MARKOV produces a spatial expansion effect around existing patches (Fig. 3). Explicit spatial expansion is not part of LCM and CA_MARKOV without filter, both of which simulate new patches of Irrigated that are not necessarily adjacent to the Irrigated at 2000. This is a reason why the output maps from LCM and CA_MARKOV without filter are more similar to each other than from either of the other two pairs of output maps.

Camacho Olmedo et al. (2013) verified that LCM simulates change at the highest ranking pixels in the transition potential maps. When CA_MARKOV uses a spatial filter, the simulation does not necessarily select the highest ranking pixels from the original suitability maps, because the spatial filter modifies the original suitability maps.

Mas et al. (2011, 2014) showed that LCM and DINAMICA tend to simulate new patches when the explanatory variables produce disjoint patches that have high transition potential. Neural networks use non-linear functions and consider interactions among variables. These machine-learning approaches allow the model to consider the fact that a single variable might have various influences across the study area (Mas et al., 2004; Aguejdad and Houet, 2008).

Other methods have been proposed for pattern validation, such as landscape metrics (Mas et al., 2010; Aguilera et al., 2011). Viana and Aranha (2010) applied LCM for the modeling and evaluation of landscape spatial patterns. CA_MARKOV has been used to analyze spatial patterns for many applications, such as to analyze: land use patterns in towns and villages (Sang et al., 2011), neighborhood effects on urbanization (Tong et al., 2012), urban growth (Mitsova et al., 2011), variation among forest management areas (Peterson et al., 2009), and the impacts of land use (XianBin et al., 2009).

Another way to focus on validation of spatial allocation is to force the simulation output to match the reference quantity of change during the validation interval. An example of simulation using reference quantities of changes has already been applied to retrospective simulation (Camacho Olmedo et al., 2008). This can be important because pattern metrics can be sensitive to the quantity of each category (Brown et al., 2013).

5. Conclusions

We recommend that the first step to validate the output of a land change model is to assess the quantity of each transition, meaning the sizes of areas of the temporal changes. Users must understand the reference transitions during the calibration and validation intervals. Then users must understand how the method for extrapolation of the quantity of each transition works. In our case study, the reference change during the calibration interval accelerated during the validation interval. Therefore, the extrapolation showed less than the reference change during validation interval. Our recommended second step to validate the outputs among land change models is to assess the variation in the allocation of changes and to determine the cause of the variation. In our case study, the 5 x 5 spatial filter made the output map from CA_MARKOV fundamentally different than the other two modeling
Fig. 5. Pattern validation during 2000–2006 by overlaying pairs of simulation outputs: LCM versus CA_MARKOV, LCM versus CA_MARKOV without filter, and CA_MARKOV versus CA_MARKOV without filter. (a) Corine persistence area. (b) Corine change area.
approaches. The 5 × 5 spatial filter causes the gain of a category to occur near where the category already existed. The other two models did not have this characteristic. The assessment of quantity and allocation of change is a helpful framework to learn how models work, and to communicate pattern validation.

Acknowledgments

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